

Homework 6  
Math 2L03

1. Use the definition of derivatives to find the derivatives of the following functions

(a)  $f(x) = x^2 + x + 4$

(b)  $g(t) = \frac{1}{t+3}$

(c)  $h(x) = \sqrt{x+4}$

2. Compute the derivative of the following functions and simplify if possible

a)  $f(x) = x^{33} + 4x^{12} + 3x^4 + \sqrt{42}x$

b)  $g(y) = \sin y + \cos y + \tan y$

c)  $h(t) = (t^4 + 6) \sin t$

d)  $f(s) = \cot s$

e)  $h(z) = \frac{\sin z}{z^3 + z^2 + 3z + 1}$

f)  $r(x) = \sqrt{x}(x^3 + 3x + 3)$

g)  $F(v) = \frac{v^3 + 3v^2 + 4}{v}$

h)  $Q(y) = (1 - y^{-1})^{-1}$

i)  $R(m) = \sqrt[5]{\frac{m^2}{\sec m}}$

j)  $S(p) = (p^3 + 2p^2 + 4)^5(3p^2 + 4)^3$

k)  $T(x) = \frac{(x-1)(x-4)}{(x-2)(x-3)}$

l)  $G(z) = \sin 3z \cos 3z$

m)  $H(t) = t^4 \sin t \cos t$

n)  $f(x) = \sqrt{\sin \sqrt{x^2 + 5}}$

o)  $g(x) = \frac{x}{\sqrt{(x^4 + 1)}}$

p)  $p(v) = (v^3 + v + 4)^5(2v^3 + 3v^2 + 2)^4$

q)  $\phi(x) = \frac{1}{\sqrt[3]{x + \sqrt[3]{x}}}$

r)  $\psi(t) = \sin^2 \left( \frac{t^3 + 1}{t^2 + 2t} \right)$

s)  $f(x) = \tan(\ln x)$

t)  $y = x^x$

u)  $h(x) = \cos(x^{\sin x})$

v)  $F(t) = e^{t \cos^2 t}$

w)  $y = \sqrt{1 + e^x \ln(x^2 + 1)}$

x)  $p(x) = [\ln(x^3 + 2x + 2)]^4$

y)  $q(x) = x^2 \log_8 \sqrt{x}$

z)  $y = x^{\sec x}$

3. Calculate the first and second derivative of the following functions

(a)  $y = \frac{x^2 - 2\sqrt{x}}{x}$

(b)  $v = \sqrt[5]{u^3} - 4\sqrt[7]{u^{11}}$

(c)  $z = \cos(x^2)$

4. Find the equation of the tangent line and the normal line to the curve at the given point

(a)  $y = x^2 + x + 3$  at  $x = 1$

(b)  $y = \sqrt{1 + x^3}$  at  $x = 2$

(c)  $y = 6 \cos x$  at  $(\frac{\pi}{3}, 3)$

(d)  $y = \sin(\sin(x))$  at  $(\pi, 0)$

5. Use logarithmic differentiation to find the derivative of

$$y = \frac{(x+1)^3(x-3)^4}{\sqrt[4]{\sin x}}.$$

6. Find  $\frac{d^{74}}{dx^{74}}(\sin x)$

7. Find the  $n$ -th derivative of  $f(x) = x^n$

8. For what values of  $x$  does the graph of  $f(x) = x^3 - 6x^2 - 15x + 4430$  have a horizontal tangent line?

9. Show that the curve  $y = 12x^3 + 10x - 3$  has no tangent line with slope 8.

10. If  $f$  and  $g$  are differentiable functions, find an expression for the derivative of each of the following functions

(a)  $y = x^n f(x)$

(b)  $y = \frac{1 + xf(x)}{\sqrt[3]{x^2}}$

(c)  $y = x^3 f(x^5)$

(d)  $y = \frac{f(x)g(x)}{f(x) + g(x)}$

11. Suppose that  $h(x) = f(x)g(x)$  and  $F(x) = f(g(x))$ , where  $f(2) = 3$ ,  $g(2) = 5$ ,  $g'(2) = 4$ ,  $f'(2) = -2$  and  $f'(5) = 11$ . Find a)  $h'(2)$  and b)  $F'(2)$

12. At what point on the curve  $y = [\ln(x+4)]^2$  is the tangent line horizontal.

13. Evaluate the following limits (By expressing the limit as a derivative)

(a)  $\lim_{x \rightarrow 1} \frac{x^{4000} - 1}{x - 1}$

(b)  $\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$